How are calculations taught at TEMIS?

Dear Parents,

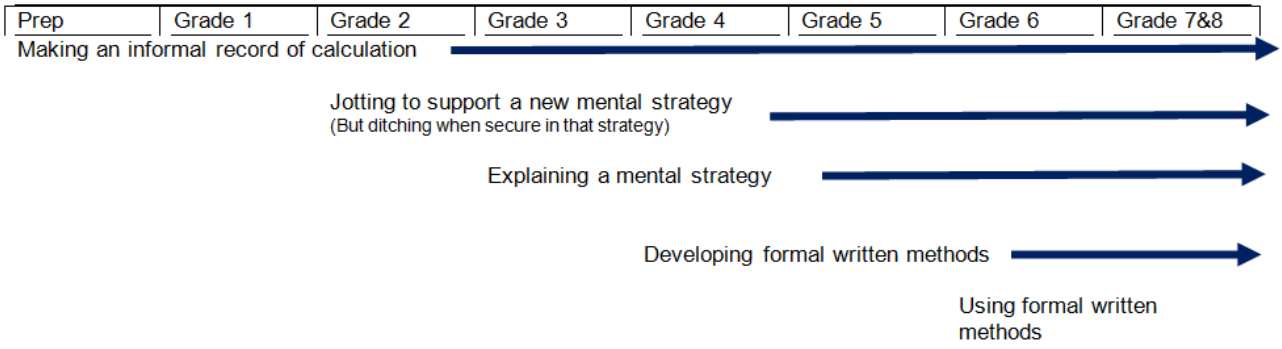
This leaflet aims to give you guidance on the different calculation methods which students are taught at TEMIS. These methods are often different to the 'sums' which you remember.

At TEMIS we focus on developing students' mental methods and understanding before teaching written algorithms. Students are encouraged to work mentally, where possible, developing personal strategies. They may use jottings or informal recordings to help support their thinking. When students are taught formal written methods they are only encouraged for calculations which cannot be solved mentally.

Written algorithms should be viewed simply as an efficient way to record processes of which students already have a thorough understanding, rather than a rote method. This approach is supported by research conducted in a range of countries.

Students are also encouraged to discuss the efficiency and suitability of different strategies. This can be extended by talking to your child about how you work things out and by asking your child to explain their thinking. Verbalising their ideas helps students consolidate their understanding and also enables you to see how confident they are with different strategies.

The table below gives some indication of the different types of written recording students are expected to use.



Throughout the document, the grade level at which each method is introduced is stated, however students are not necessarily expected to master the stage in this grade level. Within one grade level, students are often working on different stages for the same calculation, reflecting their understanding of the processes. Teachers will differentiate within each classroom to meet student needs ensuring that they consolidate an earlier stage before progressing to the next. Students will be required to demonstrate the ability to apply methods to a problem solving situation before progressing to the next stage within any of the operations. Students in Grade 7 & 8 are expected to extend the different algorithms to working with more complex decimals, ratio and algebra within the number strand.

Should you have any questions, please don't hesitate to contact your child's teacher.

The TEMIS Team

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Addition & Subtraction

Initial Learning:
Students learn to count on and back, at first with their fingers, then with number lines and hundred squares. They will do lots of practical activities which involve counting objects.

Recall of Facts:
Students need to develop recall of number bonds to 10 and then 20 (e.g. 1 + 9; 2 + 8; etc.)
They also need to be able to recall other simple addition facts such as 4 + 5 and relate this to multiples of 10 and 100 (e.g. 40 + 50).

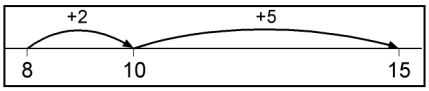
Place Value
It is also crucial that students gain an understanding of place value (how much each digit in a number is worth) and how to partition a number into ones (units), tens, hundreds etc. This helps the students manage calculations more easily.
For example:
23 + 46 can be broken down into: 20 + 40 = 60 3 + 6 = 9
60 + 9 = 69
Once students are secure in these methods, they can begin to look at written methods of calculation.

Progression in Addition

Stage 1: The empty number line (introduced Grade 1)
The steps on the way to calculating the total can be recorded on a number line. The steps often reflect a mental method.

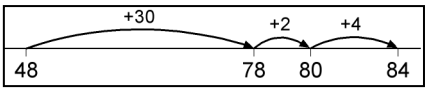
Bridging through ten

8 + 7 = 15



Partitioning

48 + 36 = 84



Stage 2: Partitioning (introduced Grade 2)
The next stage is to record mental methods using partitioning. Students add the tens and then the ones before combining these two totals. This method emphasises the value of each digit and ensures students understand what they are doing and why.

Record steps in addition using partitioning, horizontally:

47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

Partitioned numbers are then written under one another:

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +\,76 & & 70 + 6 \\ \hline & & 110 + 13 = 123 \end{array}$$

Stage 3: Expanded column method **(introduced Grade 3)**

The students can now move towards a more formal method. This layout still uses the addition of tens to tens and ones to ones separately; initially the students are allowed to do this in any order they wish but as they become confident they are asked to always add the ones first. We will always emphasise the addition of the tens as 40 + 70 and not 4 + 7.

47	47
+ 76	+ 76
13	110
110	13
123	123

Stage 4: Column method **(introduced Grade 5)**

This is the final, most compact method. Carry digits are recorded above the digits, using the words 'carry ten' or 'carry one hundred', not 'carry one'. The carry digits should be crossed off once added to avoid confusion.

It can be extended to larger numbers and decimals.

~~1 1~~	~~1 1~~	~~1 1~~
47	258	366
+76	+87	+458
123	345	824

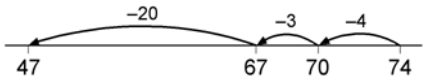
Progression in Subtraction

Stage 1: The empty number line **(introduced Grade 1)**

Steps in subtraction can be recorded on a number line.

Bridging through ten

Counting Back



Counting Up

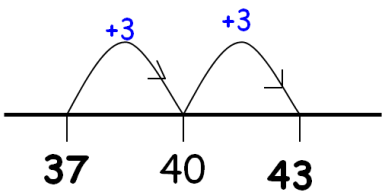
When there is a small difference between numbers, it is often easier to calculate by counting up than by counting back. It then looks similar to a number line used in addition.

43 - 37

37 + ? = 43

37 + 6 = 43

So 43-37 = 6



Stage 2: Partitioning **(introduced Grade 3)**

Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally.

74 – 27 = 74 – 20 – 7 = 54 – 7 = 47

74 – 27 = 70 + 4 – 20 – 7 = 60 + 14 – 20 – 7 = 40 + 7

Students need to be able to partition numbers in a range of ways to aid subtraction.

Eg 74 = 70 + 4 = 60 + 14

523 = 500 + 20 + 3 = 400 + 120 + 3

Stage 3: Expanded Layout, leading to column method **(introduced Grade 3)**

The expanded method leads students to the more compact method so that they understand its structure and efficiency. This is a representation of practical work students will complete using place value materials. Students should not be rushed into recording their ideas in the written form.

Partitioned numbers are written under one another:

Example: 74 − 27

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 60}\quad {\scriptstyle 14} \\ \cancel{70}+\cancel{4} \\ -\ 20+7 \\ \hline 40+7 \end{array} \qquad \begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 14} \\ \cancel{7}\ \cancel{4} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 600}\quad {\scriptstyle 130}\quad {\scriptstyle 11} \\ \cancel{700}+\cancel{40}+\cancel{1} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array} \qquad \begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 13}\ {\scriptstyle 11} \\ \cancel{7}\ \cancel{4}\ \cancel{1} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

Multiplication & Division

Initial learning
Students are encouraged to count in steps of equal sizes e.g. in twos, fives and tens. These are also the first times tables they should learn.

Times Tables
Students are introduced to other times tables. This may be done by looking at arrays and finding out facts from them.

3x4=12 12÷4=3

OR

4x3=12 12÷3=4

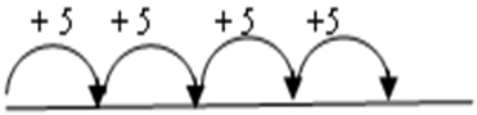
Multiplying by 10
It is essential that students can identify and apply rules for multiplying and dividing by 10, 100 and 1000.
Place value charts like this are used to demonstrate how far the digits move and in what direction when multiplying/dividing by 10/100/1000. Students are encouraged to use these charts until they are confident enough to do it without.

H	T	O	.	t	x 10
		6	.	3	
	6	3			

Application of Known Facts
Students are shown how to use known multiplication facts to help solve more difficult calculations in their heads:
e.g. If they know that 3 x 7 = 21, then they can also find:
21 ÷ 7 = 3 or 21 ÷ 3 = 7
30 x 7 = 210 or 3 x 70 = 210
300 x 7 = 2100 or 3 x 700 = 2100
3 x 0.7 = 2.1 or 0.3 x 7 = 2.1 and so on

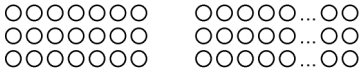
Progression in Multiplication

Stage 1: Repeated Addition (introduced Grade 2)
Students record multiplication as repeated addition on a number line.



4x5= 5+5+5+5 = 20

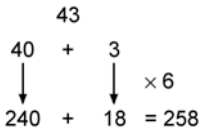
Stage 2: Mental Multiplication using partitioning (introduced Grade 2)
This method is based on the distributive law. Students should be introduced to the principle of this law for example when they use their knowledge of the 2 and 5 times-tables to work out multiples of 7:



$7 \times 3 = (5+2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$

This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product.

Either the tens or the ones can be multiplied first but it is more common to start with the tens. Informal recording might be:



Stage 3: The grid method **(introduced Grade 3)**

This method links directly to the mental method. It is an alternative way of recording the same steps.

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

Stage 4: Expanded short multiplication **(introduced Grade 4)**

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

As students become more confident, recording is reduced further.

$$\begin{array}{r} 30+8 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array} \qquad \begin{array}{l} 30\times 7=210 \\ 8\times 7=\ \ 56 \end{array}$$

When carrying out the methods students are encouraged to verbalise the true value of the digits eg. ‘thirty times seven’, rather than ‘three times seven’

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

Stage 5: Short multiplication **(introduced Grade 4)**

The recording is reduced further, with carry digits recorded above the sum. As with addition, once the carrying have been added, they should be crossed off to avoid confusion.

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for students to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

$$\begin{array}{r} \cancel{5} \\ 38 \\ \times\ 7 \\ \hline 266 \end{array}$$

Stage 6: Two-digit by two-digit products **(introduced Grade 5)**

Extend to TO × TO, asking students to estimate first. Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

The recording is reduced using the following steps:

×	20	7	
50	1000	350	1350
6	120	42	162
			1512

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 1000 \\ 120 \\ 350 \\ 42 \\ \hline 1512 \end{array} \qquad \begin{array}{l} 50\times 20=1000 \\ 6\times 20=\ \ 120 \\ 50\times 7=\ \ 350 \\ 6\times 7=\ \ 42 \end{array}$$

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 1120 \\ 392 \\ \hline 1512 \end{array} \qquad \begin{array}{l} 56\times 20 \\ 56\times\ \ 7 \end{array}$$

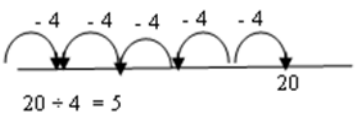
56 × 27 is approximately 60 × 30 = 1800.

Grid Method | Expanded column method | Compact method

Progression in Division

Stage 1: Repeated subtraction **(introduced Grade 3)**

Students use an empty number line to demonstrate repeated subtraction



Stage 2: Short Division 'Expanded' method for TO÷O a HTO ÷ O –"chunking" (introduced Grade 4)

It should be noted that the term **short division** is used to describe division by a single digit number, with **long division** used to describe division by a two digit number.

97 ÷ 9

9)97

− 90 9×10

7

Answer 10 r 7

This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.

It is an extension of repeated subtraction, since multiples of the divisor are subtracted.

This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially students subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.

6)196

− 60 6×10

136

− 60 6×10

76

− 60 6×10

16

− 12 6× 2

4 32

Answer 32 r 4

Stage 3:Mental division using partitioning (introduced Grade 5)

One way to work out TO÷ O mentally is to partition TO into a multiple of the divisor plus the remaining ones, then divide each part separately.

84

70 + 14

÷ 7 ÷ 7

10 + 2 = 12

Another way to record is in a grid, with links to the grid method of multiplication.

As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'

×		
7	70	14

→

×	10	2
7	70	14

10 + 2 = 12

Stage 4: Short division of TO ÷ O (introduced Grade 5)

81÷3 = (60 +21) ÷3 = (60÷3) +(21÷3)

= 20 +7 = 27

The short division method is recorded like this:

$$3\overline{)60+21}\;\;20+7$$

This is then shortened to:

$$3\overline{)8\,{}^{2}1}\;\;27$$

Stage 5 : Short division of HTO÷ O (introduced Grade 5)

The short division method is recorded like this:

$$3\overline{)290+1}=3\overline{)270+21}\;\;90+7$$

This is then shortened to:

$$3\overline{)2\,9\,{}^{2}1}\;\;9\,7$$

Stage 6: Long division – expanded method - "chunking" (introduced Grade 6)

This method builds on the expanded method for short division.

24)560

-240 24 x 10

320

-240 24 x 10

80

- 72 24 x 3

8

Answer 23 r 8

Stage 7: Long division (introduced Grade 6)

This is the conventional method of recording long division.

$$\begin{array}{r} 23 \\ 24\overline{)\,560} \\ -480 \\ \hline 80 \\ -72 \\ \hline 8 \end{array}$$

Answer: 23 R 8

How should I do this question?

When faced with a calculation problem, encourage your child to ask…

- Can I do this in my head?
- Could I do this in my head using drawings or jottings to help me?
- Do I need to use a written method?
- Should I use a calculator?

Also help your child to estimate and then check the answer.
Encourage them to ask…

Is the answer near your estimate?

Does the answer make sense?

Useful Websites

http://www.bbc.co.uk/schools/ This site from the BBC (UK) has lots of flash games, worksheets and interactive activities.
Reception refers to Prep at TEMIS, KS1 refers to Grades 1-2, KS2 refers to Grades 3-6

http://www.youcandomaths.com.au/maths-games.php from The Australian Association of Mathematics Teachers.

Below are websites from other schools/ organisations with lots of activities and games aiming to develop students' recall of facts, mental skills and ability to calculate.

http://primaryhomeworkhelp.co.uk

http://www.topmarks.co.uk/EducationalGames.aspx

http://www.amblesideprimary.com/ambleweb/numeracy.htm

http://www.copacabana-p.schools.nsw.edu.au/Get_Smart_Pages/Get_Smart.htm

http://resources.woodlands-junior.kent.sch.uk

References

This document is based on the TEMIS written calculation policy - a document for teachers. The original document was written using a range of resources and publications from different countries and educational organisations. These included the IEA Curriculum, Curricula from Australia (NSW, Queensland, Victoria & Tasmania), National Numeracy Strategy (UK), Teaching Written Calculations - Guidance for teachers at Key Stage One and Two (QCA, UK), Principles & Standards for Schools - US Council of Mathematics Teachers.